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HEAT EXCHANGE BETWEEN A SELECTIVELY EMITTING LIQUID AND A LAMINAR GAS FLOW IN THE PRESENCE OF AN EXTERNAL SOURCE OF RADIATION

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An investigation is conducted in the solution of a number of practical problems of the radiative and combined heat exchange in nonuniform systems having widely different physical properties. The processes of thermal interaction between the ocean and the atmosphere have been treated in the paper [1], the effect of thermal radiation on the melting and solidification of semitransparent crystals has been investigated in [2], the flow of a selectively emitting gas around the lateral surface of an object evaporating under the action of radiative heating has been discussed in [3], and heat transfer from a jet to the molten mass of glass in a glassmaking furnace tank has been investigated in [4]. The radiative and combined heat exchange between a selectively emitting liquid and a transparent heat-conducting laminar gas flow in the case of a specified external thermal radiation field is discussed in this paper. The energy conservation equations are set up taking into account the heat transfer by radiation, convection, and molecular thermal conduction. A differential approximation is used in calculating the values of the radiation fluxes. A system of fundamental computational equations is solved by the method of finite differences and iterations and by the Runge–Kutta method. The results of the calculations are presented in the form of graphs.

CONVENTIONAL NOTATION

$\mathrm{Bo} = d\rho c_p/\sigma(T_0 - T_m)^3$ is the Boltzmann number; $\mathrm{Iw} = \sigma(T_0 - T_m)^3 a/\lambda$ is the Ivanov number; $\mathrm{Bu}_\lambda = \varkappa_\lambda a$ is the Bouguer number; $\mathrm{Re} = da/\nu$ is the Reynolds number; $\mathrm{Bi} = \alpha a/\lambda$ is the Biot number; $\theta = (T - T_m)/(T_0 - T_m)$ is the dimensionless temperature; U and V are the longitudinal and transverse dimensionless velocity components, respectively; U_0 is the dimensionless velocity of the unperturbed gas flow; $P = p/\rho d^2$ is the dimensionless pressure; $\mu = \mu_T/\mu_{T_0}$ is the dimensionless dynamic viscosity coefficient; $E_{T,\lambda} = E^0_{T,\lambda}/\sigma(T_0 - T_m)^4$ is the dimensionless energy density of the radiation from an absolutely black body; q_λ is the dimensionless radiation flux; q_T is the dimensionless flux of heat transported by conduction; $q_n = q_T + q$ is the dimensionless net heat

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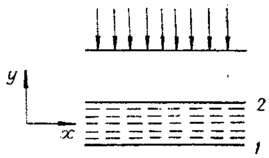

Fig. 1

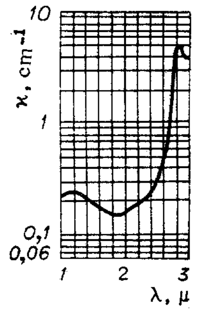

Fig. 2

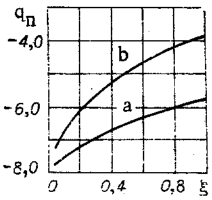

Fig. 3

flux; η and ξ are the transverse and longitudinal dimensionless coordinates, respectively; $I_\lambda(s)$ is the intensity of the radiation propagating in the direction s; E_λ^+ and E_λ^- are the surface densities of the opposed radiation fluxes propagating in the positive and negative directions of the normal to the bounding surface; T is the absolute temperature; T_0 is the average temperature of the liquid at the cross section $x=0$; T_m is the temperature of the surrounding medium; ε_λ is the degree of blackness of the plate; n_λ is the index of refraction; ρ is the density of the material; r_λ is the reflection coefficient; c_p is the specific heat at constant pressure; λ is the thermal-conductivity coefficient; d is the rate of movement of plate 2 (Fig. 1); l is the distance from the leading edge; ν is the kinematic viscosity coefficient; $E^0_{T,\lambda}$ is the energy density of the radiation from an absolutely black body; a is the thickness of the layer of liquid; $\varkappa_\lambda$ is the volume absorption coefficient of radiation; p is the pressure; c_λ is the speed of light; u_λ is the volume energy density of the radiation; α is the heat transfer coefficient; σ is the Stefan–Boltzmann constant; c is the grid function; h is the grid spacing; and β_λ is the scattering coefficient.

Quantities referring to the liquid are marked by the index 1, and those referring to the gas by the index 2.

The physical model employed is presented in Fig. 1. A layer of highly viscous liquid is contained between the fixed plate 1 and the plate 2, which is moving at constant velocity. Gas irradiated from outside by a thermal radiation flux having a wavelength-dependent density flows around plate 2.

It is necessary to calculate the distribution of the temperatures, velocities, and thermal fluxes in the liquid and the gas.

The problem was solved on the assumption of the validity of the hypothesis of thermodynamic equilibrium. Plate 2 was assumed to be transparent and infinitely thin. The radiation fluxes were assumed to be locally uniform. The dependence of the dynamic viscosity coefficient of the liquid on the temperature was described by the exponential formula

$$\mu_T = \exp 6000/T.$$

All the remaining physical properties were assumed to be independent of temperature. The motion of the liquid was assumed to be laminar, and the motion of the gas was considered far from the leading edge. Plate 1 gives off heat to its surroundings in accordance with Newton's law.

Following [5] in the calculation of the values of the radiation fluxes, we multiply the equation of transfer for spectral radiation by $\cos(s, y)d\omega_s$, and integrating over the limits of spherical solid angle 4π, we obtain an expression for the transverse component of the radiation flux vector

$$q_{y,\lambda} = -[A_\lambda/(\varkappa_\lambda + \delta_\lambda\beta_\lambda)](\partial/\partial y)(c_\lambda u_\lambda), \tag{1}$$

where

$$A_\lambda = \frac{\int\limits_{(4\pi)} \frac{\partial I_\lambda(s)}{\partial s}\cos(s, y)\,d\omega_s}{\int\limits_{(4\pi)} \frac{\partial I_\lambda(s)}{\partial y}\,d\omega_s}.$$

In the case of axisymmetric scattering indicatrices

$$\delta_\lambda = 1 - \frac{1}{4}\int\limits_0^\pi \gamma_\lambda(\vartheta)\sin 2\vartheta\, d\vartheta.$$

Differentiating the radiative energy conservation equation with respect to y, we obtain

$$\partial^2 q_{y,\lambda}/\partial y^2 = -\varkappa_\lambda \partial (c_\lambda u_\lambda)/\partial y + 4 n_\lambda^2 \varkappa_\lambda \partial E^0_{T,\lambda}/\partial y. \tag{2}$$

Using Eqs. (1) and (2), we arrive at a differential equation with respect to the values of the radiation fluxes

$$\partial^2 q_{y,\lambda}/\partial y^2 = [\varkappa_\lambda(\varkappa_\lambda + \delta_\lambda \beta_\lambda)/A_\lambda]\, q_{y,\lambda} + 4\varkappa_\lambda n_\lambda^2 \partial E^0_{T,\lambda}/\partial y. \tag{3}$$

It is necessary to supplement Eq. (3) with boundary conditions. Let $\bar{n}$ be the normal to the bounding surface which is internal with respect to the emitting volume; then

$$q_{n,\lambda} = \int_{(4\pi)} I_\lambda(s)\, \bar{n}\, d\bar{\omega}_s = E_\lambda^+ - E_\lambda^-, \tag{4}$$

$$c_\lambda u_\lambda = \int_{(4\pi)} I_\lambda(s)\, d\omega_s = \frac{\int_{(4\pi)} I_\lambda(s)\, d\omega_s}{\int_{(4\pi)} I_\lambda(s)\, |\cos(s,n)|\, d\omega_s} (E_\lambda^+ + E_\lambda^-). \tag{5}$$

If the temperature and the radiative properties of the bounding surface are specified, then

$$E_\lambda^+ = \varepsilon_\lambda n_\lambda^2 E^0_{T,\lambda} + (1-\varepsilon_\lambda) E_\lambda^-. \tag{6}$$

Upon the specification of the incident radiation density at the boundary of the system

$$E_\lambda^+ = (1 - r_\lambda) E^0_{\lambda,\mathrm{inc}}. \tag{7}$$

Using Eqs. (4)-(7) and the radiative energy conservation equation, we obtain the following boundary conditions for the differential equation (3):

$$\left(\frac{1}{\varepsilon_\lambda} - \frac{1}{2}\right) q_{n,\lambda} - \frac{1}{2\varkappa_\lambda m_\lambda}\frac{\partial q_{n,\lambda}}{\partial n} = n_\lambda^2 E^0_{T,\lambda} - \frac{4 n_\lambda^2 E^0_{T,\lambda}}{2 m_\lambda};$$

$$q_{n,\lambda} - \frac{1}{\varkappa_\lambda m_\lambda}\frac{\partial q_{n,\lambda}}{\partial n} = 2(1 - r_\lambda) E^0_{\lambda,\mathrm{inc}} - \frac{4 n_\lambda^2 E^0_{T,\lambda}}{m_\lambda},$$

where

$$m_\lambda = \frac{\int_{(4\pi)} I_\lambda(s)\, d\omega_s}{\int_{(4\pi)} I_\lambda(s)\, |\cos(s,n)|\, d\omega_s}.$$

In the case of an isotropic radiation field $A_\lambda = 1/3$, $m_\lambda = 2$. Using these values of the coefficients, neglecting scattering of the radiative energy, and changing over to dimensionless variables, we obtain the equations of differential approximation [6] for calculation of the values of the radiative fluxes in the physical model under discussion:

$$\partial^2 q_\lambda/\partial \eta^2 - 3\,\mathrm{Bu}_\lambda^2 q_\lambda = 4\,\mathrm{Bu}_\lambda n_\lambda^2 \partial E_{T,\lambda}/\partial \eta; \tag{8}$$

$$(1/\varepsilon_\lambda - 1/2) q_\lambda - (1/4\mathrm{Bu}_\lambda) \partial q_\lambda/\partial \eta = 0 \quad (\eta = 0); \tag{9}$$

$$(1/2) q_\lambda + (1/4\,\mathrm{Bu}_\lambda) \partial q_\lambda/\partial \eta = n_\lambda^2 E_{T,\lambda} - (1 - r_\lambda) E_{\lambda,\mathrm{inc}} \quad (\eta = 1). \tag{10}$$

The thermal energy conservation equations, which are set up taking account of the heat transfer by radiation, convection, and thermal conduction, are of the form

for the liquid

$$\mathrm{Bo}_1 U_1 \partial\theta_1/\partial\xi - (1/\mathrm{Iw}_1)\partial^2\theta_1/\partial\eta^2 + \partial q/\partial\eta = 0; \tag{11}$$

for the gas

$$\mathrm{Bo}_2 U_2 \partial\theta_2/\partial\xi + \mathrm{Bo}_2 V_2 \partial\theta_2/\partial\eta - (1/\mathrm{Iw}_2)\partial^2\theta_2/\partial\eta^2 = 0. \tag{12}$$

Eqs. (11) and (12) are constrained by the following boundary conditions:

at plate 1 ($\eta = 0$)

$$\partial\theta_1/\partial\eta - \mathrm{Iw}_1 q = \mathrm{Bi}_1 \theta_1; \tag{13}$$

at plate 2 ($\eta = 1$)

$$\theta_1 = \theta_2; \tag{14}$$

$$\partial\theta_1/\partial\eta = (\mathrm{Iw}_1/\mathrm{Iw}_2)\partial\theta_2/\partial\eta; \tag{15}$$

at the outer boundary of the gas ($\eta \to \infty$)

$$\partial\theta_2/\partial\eta = 0. \tag{16}$$

In Eqs. (11) and (13)

$$q = \int_0^\infty q_\lambda d\lambda. \tag{17}$$

The equations of motion and the boundary conditions for them are written

$$\mathrm{Re}\,\partial P/\partial\xi = (\partial\mu/\partial\eta)\partial U_1/\partial\eta + \mu\partial^2 U_1/\partial\eta^2; \tag{18}$$

$$U_1|_{\eta=0} = 0, \quad U_1|_{\eta=1} = 1 \tag{19}$$

for the liquid and

$$U_2 = U_0 f'(\eta^*), \; V_2 = 0{,}5\sqrt{[U_0\nu/d(\xi a + l)]}(\eta^* f' - f);$$
$$\eta^* = (\eta - 1)a\sqrt{U_0 d/\nu(\xi a + l)};$$
$$2f''' + ff'' = 0; \tag{20}$$

$$f = 0, \; f' = 1/U_0 \text{ when } \eta^* = 0, \tag{21}$$

$$f' = 1 \text{ when } \eta^* \to \infty \tag{22}$$

for the gas.

The system of Equations (8)-(19) was solved by the method of finite differences and iterations. A variable grid which compresses near the plates is used. The first and second derivatives were approximated by the difference relations

$$c_{\eta i} = (c_{i+1} - c_i)/h_i;$$

$$c_{\eta\eta,i} = (1/\hbar_i)[(c_{i+1} - c_i)/h_i - (c_i - c_{i-1})/h_{i-1}],$$

respectively, where

$$\hbar_i = 0{,}5(h_i + h_{i-1}).$$

The parabolic equations (11) and (12) and the boundary conditions (13)-(16) for them were approximated by a double-level implicit difference scheme. The systems of finite-difference equations obtained in this way were solved at each iteration by the sweep method. The boundary-value problems (20-22) was reduced with the help of the method suggested in [7] to a Cauchy problem and was solved by the Runge–Kutta method.

The wavelength dependence of the volume radiation absorption coefficient of the liquid used in the calculations is shown in Fig. 2 and corresponds to the absorption coefficient of molten window glass [8]. The en-

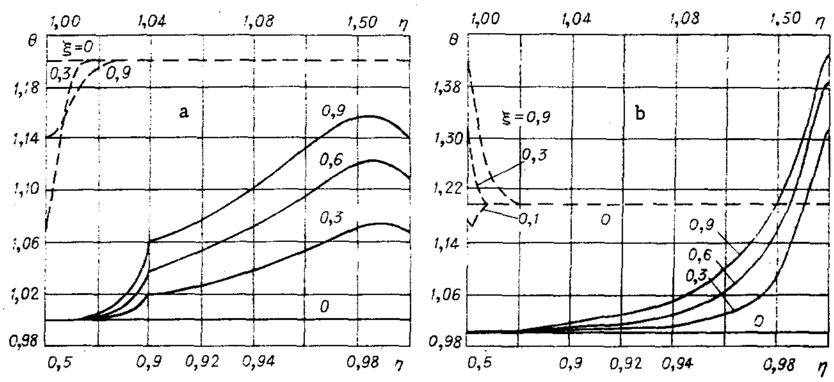

Fig. 4

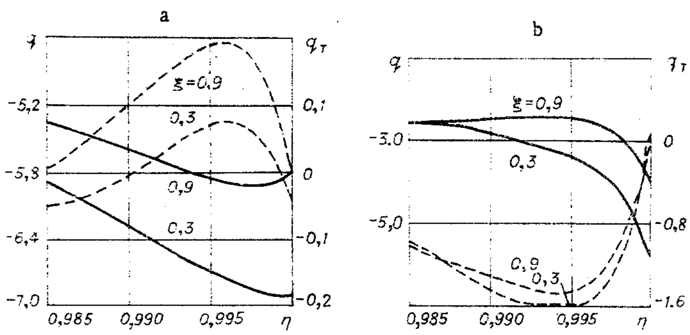

Fig. 5

TABLE 1

Wavelength interval, μ		1.0—1.2	1.2—2.0	2.0—2.4	2.4—2.65	2.65—2.8	2.8—3.0
E_i	Version 1	0.5	4.4	4.5	0.2	0.2	0.2
	Version 2	1.5	0.5	0.5	3.0	2.5	2.0

tire spectrum was broken up into a number of intervals $\Delta\lambda_i$ (i = 1, 2, ..., 6), in each of which the absorption coefficient was averaged according to the formula

$$\varkappa_{\mathrm{av}i}=\frac{1}{\Delta\lambda_i}\int\limits_{\Delta\lambda_i}\varkappa_\lambda d\lambda.$$

The results of the calculation of the temperatures and thermal fluxes for two versions of the spectral composition of the incident radiation flux are presented in Figs. 3-5 [a) first version; b) second version]. The quantities

$$E_i=\int\limits_{\Delta\lambda_i}(1-r_\lambda)E_{\lambda\,\mathrm{inc}}\,d\lambda$$

for each version are presented in Table 1 (the integrated density of the transmitted radiation $E=\sum_i E_i$ was kept unchanged). The remaining parameters had the following values:

$$\mathrm{Bo}_1=350;\ \mathrm{Bo}_2=36;\ \mathrm{Iw}_1=10;\ \mathrm{Iw}_2=307;$$
$$\mathrm{Re}=1.0;\ \partial P/\partial\xi=-6.0;\ \mathrm{Bi}=1.5;\ T_0=1300\ \mathrm{°K};$$
$$T_{\mathrm{m}}=300\ \mathrm{°K};\ d=5.0\ \mathrm{m/sec};\ U_0=10;$$
$$a=0.5\ \mathrm{m};\ l=100\ \mathrm{m}.$$

The refractive index of the liquid and the degree of blackness of the plate were assumed to be independent of wavelength and equal to n = 1 and ε = 0.1, respectively.

The sharp decrease in the net thermal flux at the surface of the liquid upon the shift from the short-wavelength to the long-wavelength region of the spectrum of the maximum of the radiative energy penetrating into the liquid (see Fig. 3) draws attention to itself. This situation is explained in the following way. Heat release to the liquid depends on the ratio of two radiation fluxes: that incident from outside and that emitted by the liquid itself. The liquid absorbs thermal radiation strongly in the long-wavelength region; therefore, in the second version only a thin surface layer of the liquid is heated up whose temperature rapidly increases (see Fig. 4b, solid curves). The energy absorbed by the surface is freely reradiated into the gaseous medium, and the net heat flux decreases. A more uniform heating of the liquid occurs in the first version (see Fig. 4a, solid curves), since the liquid is relatively transparent in the short-wavelength region of the spectrum and the thermal radiation reaches deep layers. The energy re-emitted by them is shielded by the upper layers, which results in a decrease in the outflow of thermal radiation from the liquid, and consequently to an increase in the heat release.

Thus, by varying the spectral composition of the incident radiation flux it is possible either to intensify the heat release to the liquid or to use it as a radiation screen.

We note that although the heating of the liquid occurs from above, the temperature at the surface turns out to be lower than at some depth (see Fig. 4a, b). A similar thin cooled layer is detected at the surface of the ocean and is called the cold film [1]. The heat fluxes transferred both by radiation and by molecular conduction (the solid and dashed curves, respectively, in Fig. 5a, b) affect the formation of the cold film in the physical model under discussion. The absorption of radiative energy in the deep layers of the liquid predominates over its emission in the first version of the calculation ($\partial q/\partial \eta < 0$, Fig. 5a). The upper layer of the liquid emits thermal energy freely, and starting at some temperature, the emission exceeds the absorption ($\partial q/\partial \eta > 0$ near the surface, Fig. 5a). The indicated situation results in a decrease in the temperature of the upper layer of the liquid in comparison with the temperature of the deep layers. The subsequent heating of the surface is carried out only by molecular conduction from the lower-lying layers of the liquid and from the gas (the quantities q_T and $\partial q_T/\partial \eta$ change their sign upon approaching the surface, Fig. 5a). As a result of the conductive outflow of heat to the liquid, the temperature in the surface layer of the gas becomes lower than the temperature of the undisturbed gas flow (the dashed curves in Fig. 4a). In the second version the principal portion of the energy of the incident radiation is absorbed by a thin surface layer of the liquid whose temperature increases and becomes higher than the temperature of the advancing gas flow.

In connection with this circumstance a conductive outflow of heat from the liquid to the gas begins which results in the formation of a cold film. A change in the sign of the conductive component of the thermal flux causes the formation of thermal waves in the surface layer of the gas (dashed curves in Fig. 4b). The deep layers of the liquid, having attained a specified temperature, begin to lose thermal energy by emission ($\partial q/\partial \eta > 0$, Fig. 5b), and their subsequent heating is accomplished by the conductive supply of heat from the hot surface layers.

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